Mixing and segregation rates in sheared granular materials

Laura A. Golick and Karen E. Daniels[*]

Department of Physics, North Carolina State University, Raleigh, North Carolina 27695, USA
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The vertical size segregation of granular materials, a process commonly associated with the Brazil-nut effect, has generally been thought to proceed faster the greater the size difference of the particles. We experimentally investigate sheared dense bidisperse granular materials as a function of the size ratio of the two species and find that the mixing rate at low confining pressure behaves as expected from percolation-based arguments. However, we also observe an anomalous effect for the resegregation rates, wherein the segregation rate is a nonmonotonic function of the particle size ratio with a maximum for intermediate particle size ratio. Combined with the fact that increasing the confining pressure significantly suppresses both mixing and segregation rates of particles of sufficiently dissimilar size, we propose that the anomalous behavior may be attributed to a species-dependent distribution of forces within the system.

Accurate knowledge of the rate at which granular materials segregate by size under shear [1,2] is significant for such applications as avalanche hazard prediction and the design of industrial particle separation chutes. One of the most common segregation phenomena, the Brazil-nut effect [3], is broadly observed and has been associated with a variety of proposed mechanisms [4]. For dense shear flows, kinetic sieving theory in various forms [5–7] has been the most promising. These theories rely on statistical arguments which quantify the creation of voids through shear: smaller particles preferentially fall into these voids in a percolationlike fashion. Therefore, it is expected that the larger the difference in particle sizes, the quicker this process will happen. While percolation rates have previously been measured in a quasi-two-dimensional experiment [8], there is to date no fundamental understanding of the size dependence and pressure dependence for true three-dimensional flows nor is it known in which regimes kinetic sieving is the dominant effect.

We investigate the mixing and subsequent resegregation of a dense granular material initially configured so that a layer of small particles is placed above an equal volume of large particles within an annular shear cell. Under shear from the bottom plate, the small particles migrate to the bottom and the large particles correspondingly migrate to the top, as is expected for particles of otherwise identical material [2]. We measure the mixing and segregation rates as a function of particle size ratio and confining pressure and find that the mixing rate is consistent with kinetic sieving models for approximately hydrostatic confining pressure. However, the segregation rates are observed to be nonmonotonic in particle size ratio, in contrast with kinetic sieving theory, and strongly depend on the confining pressure. Below, we quantify these rates and interpret them in light of the heterogeneous force-transmission properties of granular materials.

Our experimental cell is an annulus which confines the particles between a top plate free to move vertically and a rotating bottom plate as shown in Fig. 1(a). The radius of the

inner wall of the annulus is 25.5 cm and the width of the particle-filled channel is 3.8 cm. Both the top and bottom plates are lined with rubber to increase their friction coefficient, while the sidewalls are bare aluminum. The bottom plate has a rotation period of 20.4 s (frequency $f=49.0$ mHz) creating a shear band which extends a few particle diameters [9] into the cell. We adjust the confining pressure of the cell via two techniques: weighting the top plate to increase the pressure or partially suspending the top plate from springs to reduce the pressure on the granular aggregate.

Each experimental run begins in an initial state consisting of a layer of small particles (mass 2 kg) over a layer containing an equal mass (volume) of large particles. This initial configuration is shown in Fig. 1(b). Both large and small particles are glass spheres of identical density and we can therefore use mass and volume interchangeably. The small particles are a single size for each run, with diameter d_S ranging from 1.5 to 5.0 mm; large particles are fixed at diameter $d_L=6$ mm for all runs. Once shear begins, the small particles filter downward through the large particles resulting in a mixed state such as the one shown in (c). Eventually, nearly all of the large particles have reached the top of the cell (d). This experimental protocol allows us to examine both the mixing of small and large particles and the subsequent resegregation of the mixture.
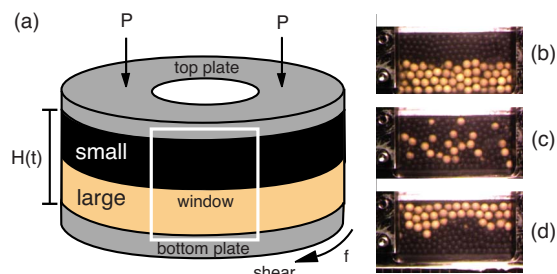

FIG. 1. (Color online) (a) Schematic of experimental apparatus (not to scale) showing initial configuration of particles within the annulus. Sample images taken at window for $d_S=4$ mm (dark particles) and $d_L=6$ mm (light particles): (b) initial configuration, (c) mixed state, and (d) final resegregated state.
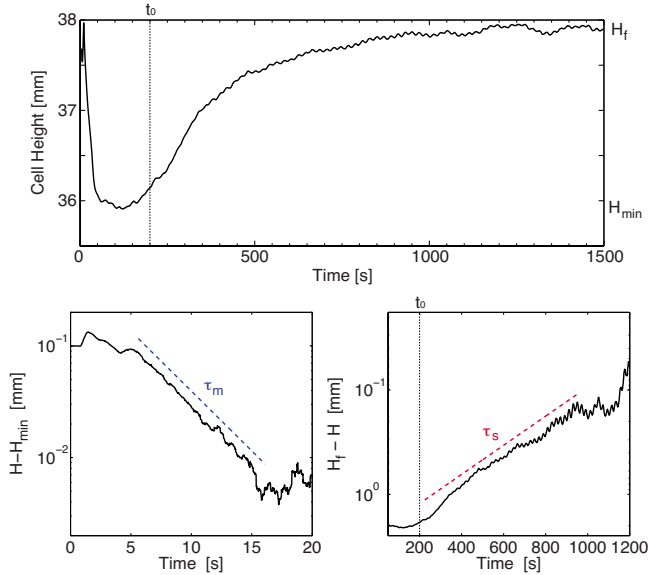
*kdaniel@ncsu.edu

FIG. 2. (Color online) Sample cell height data for $r=2/6$ and $\widetilde{P}=0.36$. An averaging window of 0.5 s was used to smooth the raw signal. (a) Cell height $H(t)$ with values $H_{\min}$, H_f, and t_0 marked. (b) Magnified portion of $H(t)-H_{\min}$ showing fit to determine mixing time scale τ_m. (c) Magnified portion of $H_f-H(t)$ showing fit to determine segregation time scale τ_s.

The view at the outside wall is not necessarily representative of the bulk behavior particularly for mixtures of very different sized particles. Therefore, to measure the average behavior of the whole system, we monitor the height $H(t)$ of the top plate as shown in Fig. 2(a). When shear begins, the system initially expands due to Reynolds dilatancy. As small particles fill the gaps between the large particles during mixing, the overall cell height quickly decreases. When the particles are well mixed, the aggregate takes up the least total space and falls to a height $H_{\min}$. As resegregation occurs, the system redilates to a final height H_f. We measure the time scale τ_m for this mixing process by fitting the function $H(t)-H_{\min} \propto e^{-t/\tau_m}$ to the decrease in the height of the cell. As the particles begin to resegregate, the large particles rise through the mix and ultimately end up in a layer above the small particles. For this process, we define a segregation time scale τ_s by fitting a function of the form $H_f-H(t) \propto e^{-(t-t_0)/\tau_s}$, where t_0 is chosen to be after the minimum $H_{\min}$. Representative fits for τ_m and τ_s are shown in Figs. 2(b) and 2(c).

As a consequence of fixing the mass (volume) of the particles, the total number of particles varies with particle size ratio $r=d_S/d_L$. Therefore, we scale the cell height by an appropriate mean particle diameter δ such that H/δ represents the height of the cell measured in particle diameters. This scaling reflects the fact that the number of net layer transitions made by a particle scales with the number of particle layers δ present in the system. For each (d_S,d_L) pair, we define δ via the relationship

$$\frac{2}{\delta} = \frac{1}{d_S} + \frac{1}{d_L}. \tag{1}$$

Using δ, we can compare nondimensionalized mixing (Ω_m) and segregation (Ω_s) rates among runs with different r:

$$\Omega_{m,s} = \frac{H}{\delta f \tau_{m,s}}, \tag{2}$$

where f is the constant rotation frequency of the bottom plated.

By varying the confining pressure, we explore a regime in which the imposed pressure is either greater or less than the mean "hydrostatic" pressure due to the weight of the particles on the bottom plate. Note that because of the roughly square cross-sectional area of the experimental cell, the Janssen effect is small and consequently the internal pressure increases with depth. In addition, the local pressure spatially varies due to the presence of force chains [10]. Since the weight of the particles provides a natural unit for the pressure, we scale the effective weight of the top plate by the weight of the particles and report dimensionless $\widetilde{P}$ defined as

$$\widetilde{P} = \frac{m_{\mathrm{p}}g + Mg - k\Delta x}{m_{\mathrm{g}}g}, \tag{3}$$

where $m_{\mathrm{p}}=15.42$ kg is the mass of the top plate, $m_{\mathrm{g}}=4$ kg is the total mass of the particles, M is the added compressive mass (if present), and $k\Delta x$ is the average upward force from the supporting springs (if present). We explore values of $\widetilde{P}$ from 0.25 to 1.48; typical variation within a single run is $\pm 8 \times 10^{-3}$ due to the contraction/extension of the supporting springs. We add mass $M=0$ to 4.5 kg to increase compression; the smallest $\widetilde{P}$ is achieved by adjusting the length of the spring supports.

We measure the mixing and segregation rates for six different particle size ratios with $\widetilde{P}=0.36$ (at least five runs each) and at six different pressures for $r=2/6$ and $r=5/6$ (at least three runs each). Figure 3 depicts the mixing and segregation time scales and rates as a function of particle size ratio with pressure and f held constant. We observe that the mixing rate Ω_m decreases as particles become more similar in size $(r \rightarrow 1)$. This corresponds to the expected kinetic sieving behavior [1,6–8] whereby small particles filter down through a fluctuating "sieve" of large particles. The smaller r is, the more likely the small particles are to find voids to fall into.

In contrast, we observe that the resegregation process takes longer for both small and large r, as shown in Fig. 3, whether measured as elapsed time or a rate scaled by δ. A maximum segregation rate is achieved near $r=3/6$: further reductions in the smaller particle size slow the rate at which the system resegregates.

In order to better understand this behavior, it is worthwhile to examine how Ω_s depends on the confining pressure on the system at both large and small r. As shown in Fig. 4, increasing $\widetilde{P}$ decreases Ω_m and the pressure affects contrasting particle sizes (low r) more strongly than similar particle sizes. For Ω_s at low r, this effect is even more pronounced: a fivefold increase in pressure decreases the segregation rate by a factor of 100. This strong suppression of segregation with pressure causes an inversion in the r dependence for $\widetilde{P} \gtrsim 0.5$. Pressure has little effect on either rate as $r \rightarrow 1$.
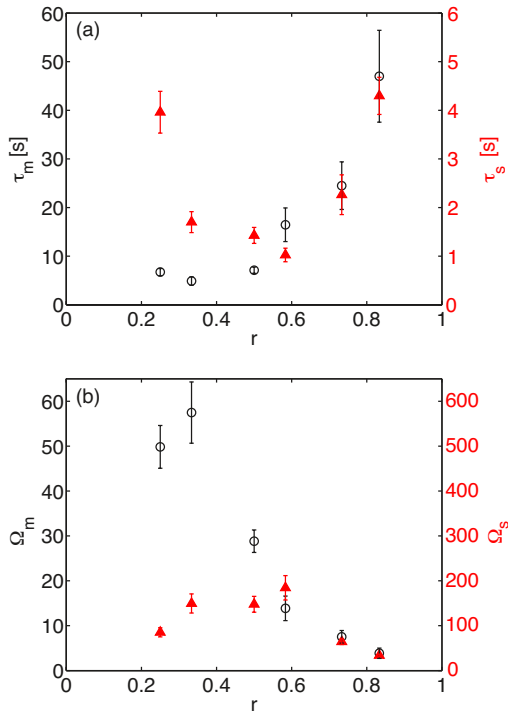
FIG. 3. (Color online) (a) Mixing time scale τ_m ($\bigcirc$, left axis) and segregation time scale τ_s ($\blacktriangle$, right axis) as function of particle size ratio r. (b) Mixing rate Ω_m ($\bigcirc$, left axis) and segregation rate Ω_s ($\blacktriangle$, right axis) as function of particle size ratio r. The error bars represent the standard error among at least five independent measurements. All data are collected at $\tilde{P}=0.36$.

These results provide three effects in need of explanation: (1) segregation rates display nonmonotonic dependence on particle size ratio, (2) contrasting particle sizes are much more sensitive to pressure than similar particle sizes, and (3) mixing rates are much faster than segregation rates. The decrease in the segregation rate for small r is particularly notable, since it is inconsistent with the predictions of kinetic sieving. The pressure sensitivity of the system suggests looking at force chains [10,11] as an important factor for all three effects.

In simulations of granular materials in two dimensions, it is observed that force chains preferentially form through the larger particles as size ratio r decreases [12–15]; this is likely related to the large particles' enhanced number of contacts. This unequal partitioning of force chains between large and small particles, were it to also be present for three-dimensional granular materials, could account for the first two effects. For small r, the presence of a large-particle-dominated force chain network at larger pressures could make it difficult for small particles to rearrange, thus slowing the segregation rate. As the particles become more similar in size (increasing r), such an imbalance would be smaller in magnitude.

Another factor that could influence the anomalously low segregation rate of the system for small r is the observed increase in packing fraction for mixtures of dissimilar particle sizes [16]. If the experiments at low r are denser, then they have less void space and this could slow their resegregation.
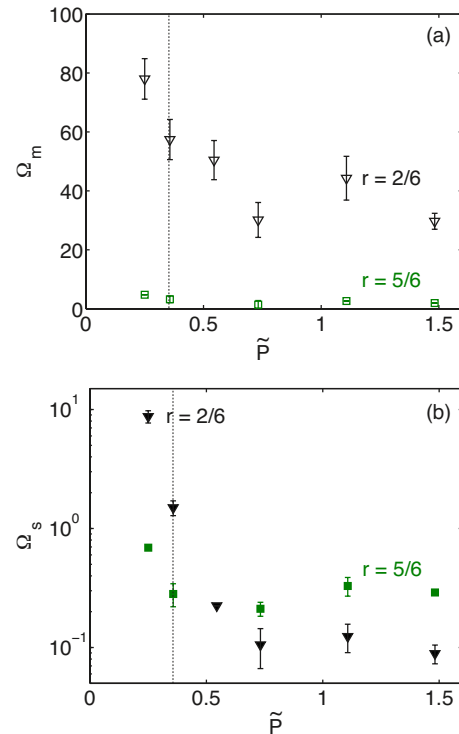

FIG. 4. (Color online) (a) Mixing rate Ω_m as a function of scaled pressure for $r=2/6$ ($\triangledown$) and $r=5/6$ ($\square$). (b) Segregation rate Ω_s as a function of scaled pressure for $r=2/6$ ($\blacktriangledown$) and $r=5/6$ ($\blacksquare$). The error bars represent the standard error among at least three independent measurements. The dashed line is $\tilde{P}=0.36$, which coincides with data from Fig. 3.

We also observe a lack of reciprocity in the mixing and segregation mechanisms: a small particle falling though a mixture of mostly large particles (mixing) does not progress at the same rate as a large particle rising through a mixture of both large and small particles (segregation). Not only is there an approximately $10\times$ difference in the associated rates (see Fig. 3), but the segregation rates are much more pressure dependent than the mixing rates. The mixing process can be more clearly associated with the void-filling mechanisms of kinetic sieving, which are apparently not strongly influenced by pressure. However, the segregation process requires large particles rising (called "squeeze expulsion" by [6]), which cannot be described by void filling.

These experiments highlight the fact that granular segregation provides a sensitive probe of how both the void space and the stress transmission influence the dynamics of the system. The pressure dependency of the results suggests that volume-based descriptors of the state of granular systems [17] should be supplemented by information on the stresses [18–22]. In the experiments described here, we are unable to measure either the void distributions or the force distributions for large-P and small-P cases, so we cannot disentangle the two effects. While local free volume distributions have recently been measured in three-dimensional systems [23], little is yet known on how such distributions are affected by pressure or shear. An improved understanding of the interplay between pressure and volume state variables will improve models of segregation.

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